Exercise set 6 - Dynamics

Notation

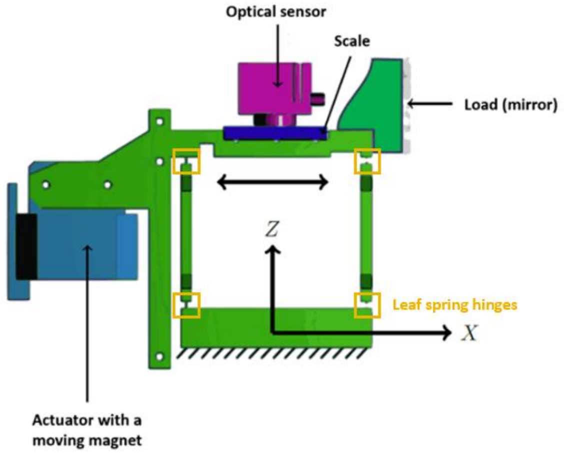
Please note that in literature, the **inertia** and the **Jacobian matrix** are sometimes denoted using similar symbols (commonly J or $\mathbf{J}$). The specific meaning should be interpreted based on context.

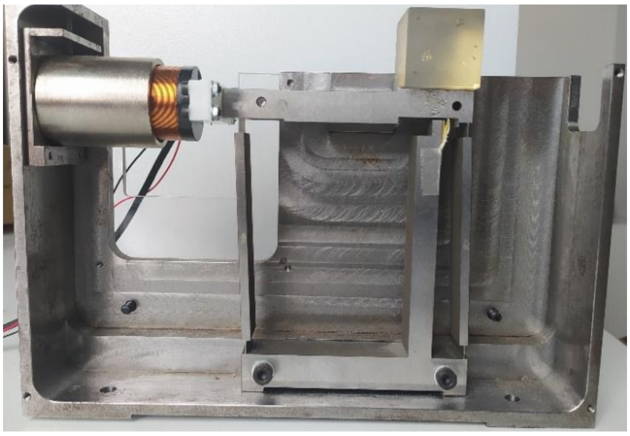
Also, to help in distinguishing them, you can check the **units and dimensions**:

- Jacobian → relates linear or angular velocities → units like m/rad, rad/rad, rad/m, m/m, dimension depends on the number of joint coordinates and degrees of freedom.
- Inertia → relates torques and accelerations → units like g·cm², typically scalar for the rotation inertia around a given axis, e.g., for the inertia of a motor along its rotation. For a rigid body, if not stated otherwise, the given inertia is considered for a rotation around the center of mass along the principal axis relevant for the motion.

Exercise 1

Consider the following flexure hinge-based guide actuated by a current-controlled DC motor. A video illustrating the movement of the mechanism is uploaded on moodle.

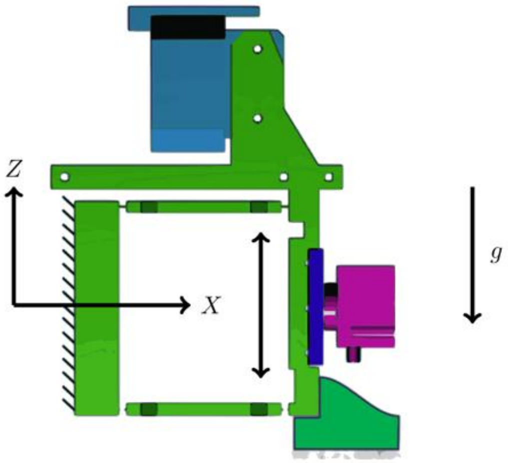




Let:

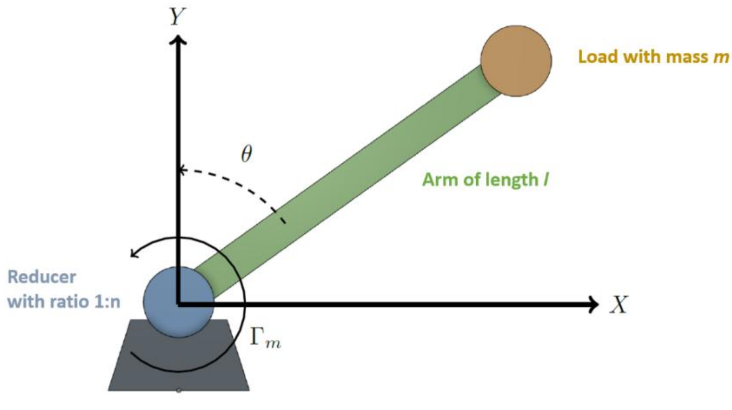
- k_f be the force constant between the active generated force and the motor current.
- m_g be the equivalent mass of the flexure hinge guide.
- m_l be the mass of the load (mirror and support).
- k_s be the equivalent linear stiffness of the leaf spring hinges.

1. Give the expression of the inverse dynamic model of this linear axis.
2. Give the expression of the direct dynamic model of this linear axis.
3. Redo questions 1 and 2 by placing the linear axis vertically and referencing the vertical axis to the point corresponding to the zero spring force. Draw the block diagram showing the inputs and output of the system.



Exercise 2

The most common rotational axis model in robotics corresponds to a rotary motor with a reducer, an arm and a load at the end. If we do not consider the couplings, all robotic arms can be represented by the model shown in the figure below.



Let:

- J_m be the inertia of the motor.
- n be the reduction ratio.

- J_a be the inertia of the arm (includes the inertia of the reducer and the coupling).
- m_a be the mass of the arm.
- l be the length of the arm.
- m be the load at the end of the arm.
- k_{vis} be the coefficient of viscosity referred to the load side.

1. Give the total moment of inertia at the axis of rotation of the load on the load side.
2. Give the expression of the inverse dynamic model of this rotational axis.
3. Give the expression of the direct dynamic model of this rotational axis.
4. How to use the previous results to size the motor needed for a given application?